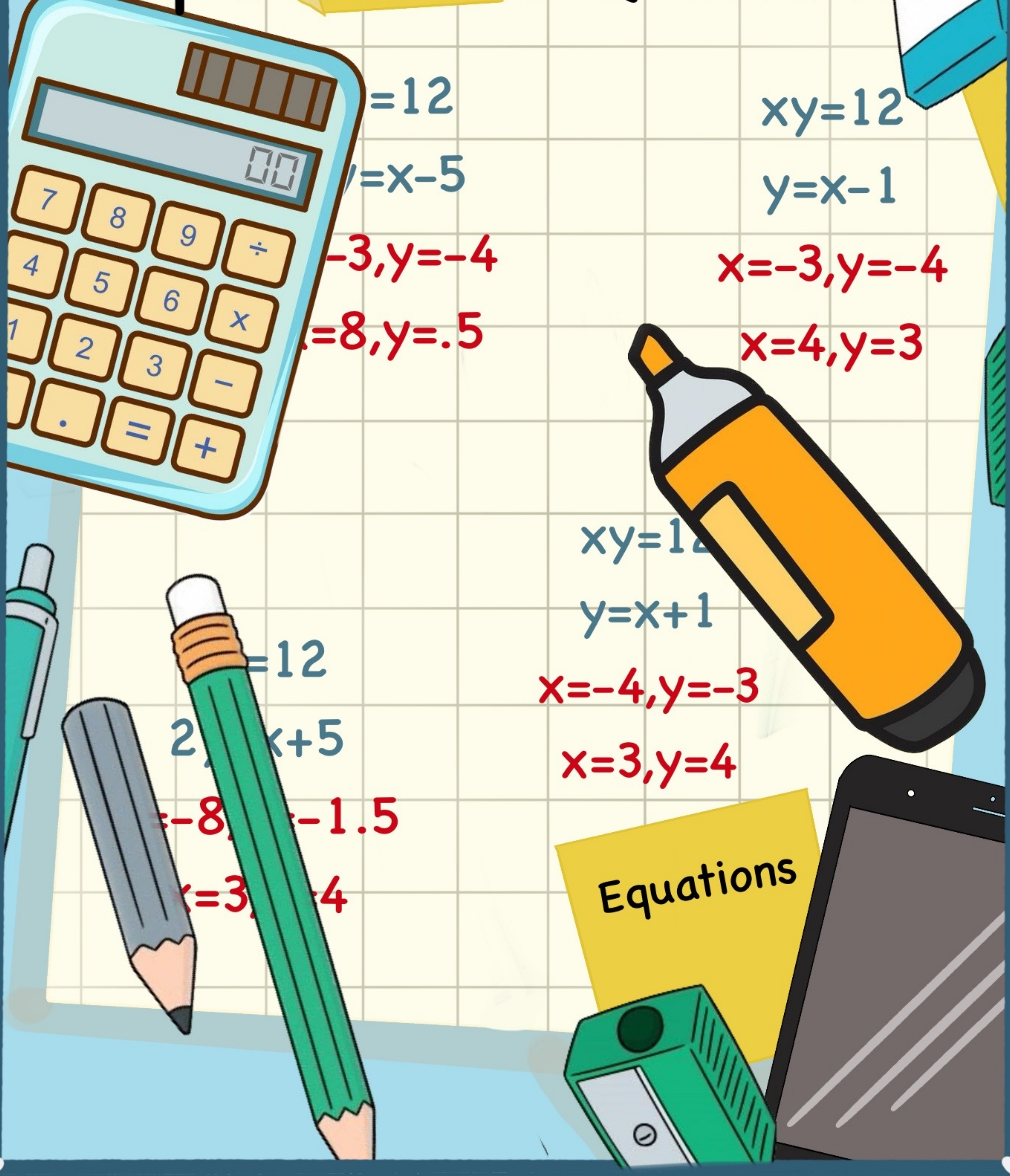
The Maths Society

Simultaneous

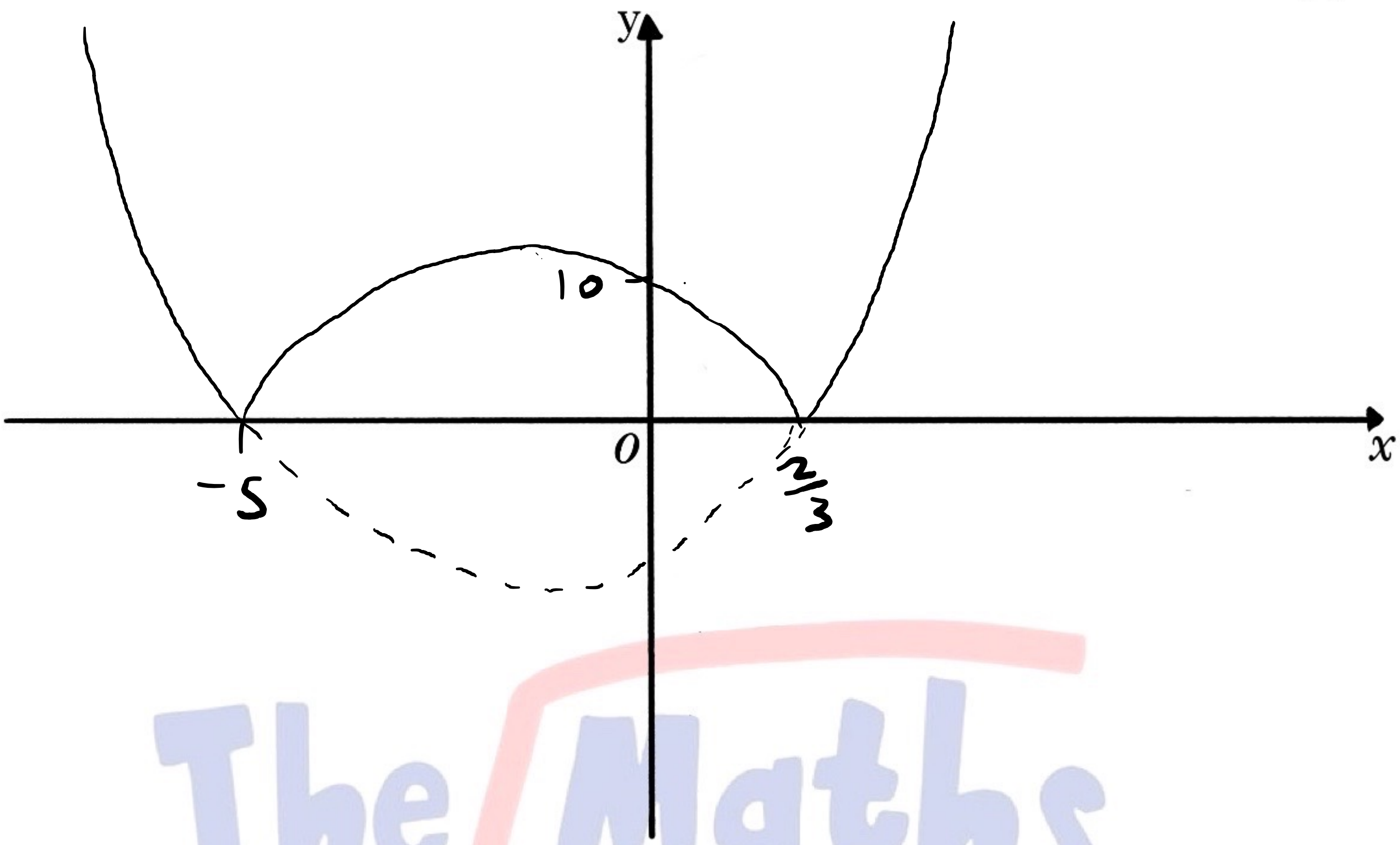
Equations and Quadratics



Chapter 2 - Simultaneous Equations and Quadratics

1.(a) On the axes, draw the graph of $y = |3x^2 + 13x - 10|$, stating the coordinates of the points where the graph meets the axes.

[4]



$3x^2 + 13x - 10 = (3x - 2)(x + 5)$

$\therefore$ roots are $x = -5, \frac{2}{3}$

(b) Find the set of values of the constant k such that the equation $k = |3x^2 + 13x - 10|$ has exactly 2 distinct roots.

[4]

midpt of x-intercept $= \dfrac{-5 + \frac{2}{3}}{2}$

$= -\dfrac{13}{6}$

$y = 3(-\frac{13}{6})^2 + 13(-\frac{13}{6}) - 10$

$= -\dfrac{289}{12}$

$|y| = \dfrac{289}{12}$ $\quad \therefore \left(-\frac{13}{6}, \frac{289}{12}\right)$

$\therefore k = 0$ or $k > \dfrac{289}{12}$

[or]

$3x^2 + 13x - 10$

$= 3\left(x^2 + \frac{13}{3}x\right) - 10$

$= 3\left[\left(x + \frac{13}{6}\right)^2 - \frac{169}{36}\right] - 10$

$= 3\left(x + \frac{13}{6}\right)^2 - \dfrac{289}{12}$

$k > \dfrac{289}{12}$ $\quad k = 0$

2.(a) Show that $2x^2 + x - 15$ can be written in the form $2(x + a)^2 + b$, where a and b are exact constants to be found.

[2]

$$2x^2 + x - 15 = 2\left[x^2 + \frac{x}{2}\right] - 15$$
$$= 2\left[\left(x + \frac{1}{4}\right)^2 - \frac{1}{16}\right] - 15$$
$$= 2\left(x + \frac{1}{4}\right)^2 - \frac{121}{8}$$

$\therefore a = \frac{1}{4}, \ b = -\frac{121}{8}$

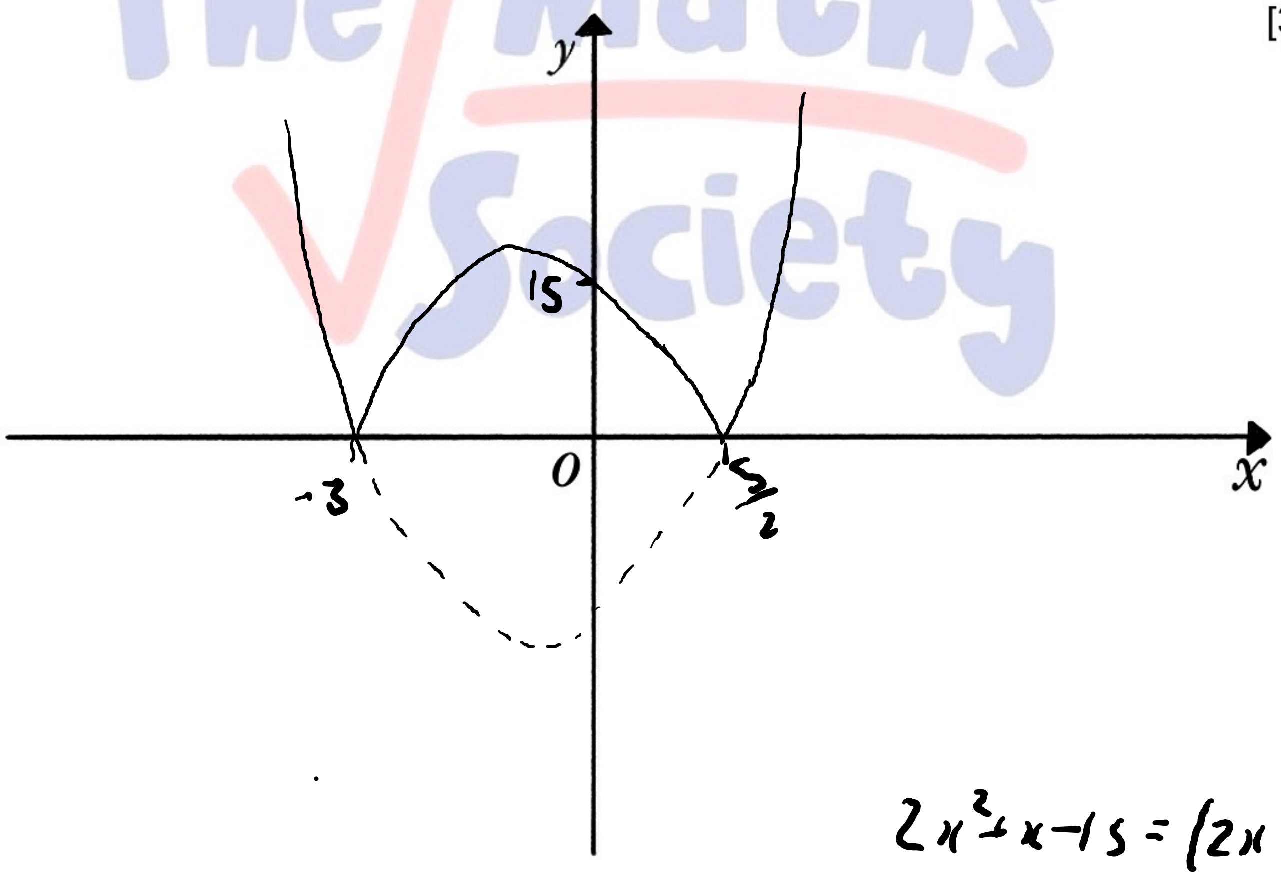
(b) Hence write down the coordinates of the stationary point on the curve $y = 2x^2 + x - 15$.

[2]

$\left(-\frac{1}{4}, -\frac{121}{8}\right)$

(c) On the axes,sketch the graph of $y = |2x^2 + x - 15|$, starting the coordinates of the points where the graph meets the coordinates axes.

[3]



$2x^2 + x - 15 = (2x - 5)(x + 3)$

(d) Write down the value of the constant k for which the equation $|2x^2 + x - 15| = k$ has 3 distinct solutions.

[1]

$k = \frac{121}{8}$

3.Solve the following simultaneous equations.

$$3y - 2x + 2 = 0$$
$$xy = \frac{1}{2}$$

[3]

$y = \frac{1}{2x}$

$\frac{3}{2x} - 2x + 2 = 0$

$3 - 4x^2 + 4x = 0$

$(2x+1)(2x-3) = 0$

$x = -\frac{1}{2}, \frac{3}{2}$

when $x = -\frac{1}{2}$, $y = \frac{1}{2(-\frac{1}{2})} = -1$

When $x = \frac{3}{2}$, $y = \frac{1}{2(\frac{3}{2})} = \frac{1}{3}$

4.DO NOT USE A CALCULATOR IN THIS QUESTION.

Find the x-coordinates of the points where the line $y = 3x - 8$ cuts the curve $y = 2x^3 + 3x^2 - 26x + 22$.

[5]

$$2x^3 + 3x^2 - 26x + 22 = 3x - 8$$

$$2x^3 + 3x^2 - 29x + 30 = 0$$

$$\begin{array}{r}
2x^2 + 7x - 15 \\
x - 2 \enclose{longdiv}{2x^3 + 3x^2 - 29x + 30} \\
2x^3 - 6x^2 \\
\hline
7x^2 - 29x \\
7x^2 - 14x \\
\hline
-15x + 30 \\
-15x + 30 \\
\hline
0
\end{array}$$

$$(2x^2 + 7x - 15) = (2x - 3)(x + 5)$$

$$\therefore x = -5, \tfrac{3}{2}, 2$$

when $x = -5$, $y = 3(-5) - 8$
$= -23$ $(-5, -23)$

when $x = \frac{3}{2}$, $y = 3(\frac{3}{2}) - 8$
$= -\frac{7}{2}$ $(\frac{3}{2}, -\frac{7}{2})$

when $x = 2$, $y = 3(2) - 8$
$= -2$ $(2, -2)$

5.(a) Write $3x^2 + 15x - 20$ in the form $a(x + b)^2 + c$ where a, b and c are rational numbers.

[4]

$$3x^2 + 15x - 20 = 3\left[x^2 + 5x\right] - 20$$

$$= 3\left[\left(x + \frac{5}{2}\right)^2 - \frac{25}{4}\right] - 20$$

$$= 3\left(x + \frac{5}{2}\right)^2 - \frac{155}{4}$$

$\therefore a = 3$, $b = \frac{5}{2}$, $c = -\frac{155}{4}$

(b) State the minimum value of $3x^2 + 15x - 20$ and the value of x at which it occurs.

[2]

$\left(-\frac{5}{2}, -\frac{155}{4}\right)$

(c) Use your answer to **part (a)** to solve the equation $3y^{\frac{2}{3}} + 15y^{\frac{1}{3}} - 20 = 0$, giving your answers correct to three significant figures.

[3]

Let $y^{1/3} = x$

$$3\left(x + \frac{5}{2}\right)^2 = \frac{155}{4}$$

$$x + \frac{5}{2} = \pm \frac{\sqrt{465}}{6}$$

$$x = -\frac{5}{2} \pm \frac{\sqrt{465}}{6}$$

$$y = x^3$$

$$= -226, 1.31$$

6.Solve the following simultaneous equations.

$$x + 5y = -4$$
$$3y - xy = 6$$

[5]

$x = -5y - 4$

$3y - y(-5y - 4) = 6$

$3y + 5y^2 + 4y = 6$

$5y^2 + 7y - 6 = 0$

$(5y - 3)(y + 2) = 0$

$y = -2, \frac{3}{5}$

when $y = -2$, $x = -5(-2) - 4$
$= 6$

when $y = \frac{3}{5}$, $x = -5(\frac{3}{5}) - 4$
$= -7$

7.Solve the following inequality.

$$(2x + 3)(x - 4) > (3x + 4)(x - 1)$$

[5]

$2x^2 - 8x + 3x - 12 > 3x^2 - 3x + 4x - 4$

$x^2 + 6x + 8 < 0$

$(x + 4)(x + 2) < 0$

$-4 < x < -2$

8. Find the possible values of k for which the equation $kx^2 + (k+5)x - 4 = 0$ has real roots.

[5]

For quadratic to have real roots:

$b^2 - 4ac \geq 0$

$\Rightarrow (k+5)^2 - 4k(-4) \geq 0$

$k^2 + 10k + 25 + 16k \geq 0$

$k^2 + 26k + 25 \geq 0$

$(k+25)(k+1) \geq 0$

$k \leq -25 \quad k \geq -1$

9.(a) Find the range of value of x satisfying the inequality $(5x - 1)(6 - x) < 0$.

[2]

$30x - 5x^2 - 6 + x < 0$

$5x^2 - 31x + 6 > 0$

$(5x - 1)(x - 6) > 0$

$x < \frac{1}{5} \quad x > 6$

(b) Show that the equation $(2k + 1)x^2 - 4kx + 2k - 1 = 0$, where $k \neq -\frac{1}{2}$, has distinct, real roots.

[3]

For quadratic to have distinct, real roots:

$b^2 - 4ac > 0$

$= (-4k)^2 - 4(2k+1)(2k-1)$

$= 16k^2 - 4(4k^2 - 1)$

$= 16k^2 - 16k^2 + 4$

$= 4 > 0$

$\therefore$ distinct, real roots.

10.Find the values of k such that the line $y = 9kx + 1$ does not meet the curve $y = kx^2 + 3x(2k + 1) + 4$.

[5]

$kx^2 + 3x(2k+1) + 4 = 9kx + 1$

$kx^2 + 3x(1-k) + 3 = 0$

$b^2 - 4ac < 0$ if lines do not meet

$[3(1-k)]^2 - 4k(3) < 0$

$9(1-2k+k^2) - 12k < 0$

$9k^2 - 30k + 9 < 0$

$3k^2 - 10k + 3 < 0$

$(3k-1)(k-3) < 0$

$\frac{1}{3} < k < 3$